Test 12-16

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Without Templates

Theorem 1:*an xn* **+ ... +** *a1 x* **+** *a0* is *O***(** *xn* **)** for any real numbers *an* , ..., *a0* and any nonnegative number *n* **.**

With Templates

an xn + ... + a1 x + a0 is O(xn) for any real numbers an , ..., a0 and any nonnegative number n .

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Without Templates

Theorem 1: calling makeLayer with valid inputs (a list of weights and two non-zero natural numbers) returns a valid layer recognized by isLayer. (“layers.lisp”)

Theorem 2: calling makeNetwork with valid inputs (a list of non-zero natural numbers and a list of weights) returns a valid network recognized by isNetwork. (“networks.lisp”)

With Templates

calling makeLayer with valid inputs (a list of weights and two non-zero natural numbers) returns a valid layer recognized by isLayer. (“layers.lisp”)

calling makeNetwork with valid inputs (a list of non-zero natural numbers and a list of weights) returns a valid network recognized by isNetwork.

("networks.lisp")

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Without Templates

Theorem 4: *an xn* **+ ... +** *a1 x* **+** *a0* is θ**(** *xn* **)** for any real numbers *an* , ..., *a0* and any nonnegative number *n* **.**

Let *f*(*x*) and *g*(*x*) be functions from a set of real numbers to a set of real numbers.

Then

1. If $f(x)/g(x) = 0$, then *f*(*x*) is *o*(*g*(*x*)) . Note that if *f*(*x*) is *o*(*g*(*x*)), then *f*(*x*) is *O*(*g*(*x*)).

2. If $f(x)/g(x) = \infty$, then *g*(*x*) is *o*(*f*(*x*)) .

3. If $f(x)/g(x) < \infty$, then *f*(*x*) is θ(*g*(*x*)) .

4. If $f(x)/g(x) < \infty$, then *f*(*x*) is *O*(*g*(*x*)) .

For example,

$(4x^3 + 3x^2 + 5)/(x^4 – 3x^3 – 5x -4)$

= (*4/x* **+** *3/x 2* **+** *5/x 4* **)/(** *1* **-** *3/x* **-** *5/x 3* **-** *4/x 4* **) =** *0* **.**

Hence

(*4x3* **+** *3x2* **+** *5* **)** is *o*(*x4* **-** *3x3* **-** *5x* **-** *4* **),**

or equivalently, **(***x4* **-** *3x3* **-** *5x* **-** *4* **)** is ω(*4x3* **+** *3x2* **+** *5* **) .**

Let us see why these rules hold. Here we give a proof for 4. Others can be proven similarly.

Proof: Suppose $f(x)/g(x) = C_1 < \infty$.

By the definition of limit this means that

$\forall \varepsilon > 0$, $\exists n_0$ such that $|f(x)/g(x) - C1| < \varepsilon$ whenever $x > n_0$

Hence $-\varepsilon < f(x)/g(x) - C1 < \varepsilon$

Hence $-\varepsilon + C1 < f(x)/g(x) < \varepsilon + C1$

In particular $f(x)/g(x) < \varepsilon + C1$

Hence $f(x) < (\varepsilon + C1)g(x)$

Let $C = \varepsilon + C1$, then $f(x) < Cg(x)$ whenever $x > n_0$.

Since we are interested in non-negative functions *f* and *g*, this means that
$|f(x)| \leq C\, |g(x)|$

Hence $f(x) = O(g(x))$.

With Templates

an xn + ... + a1 x + a0 is θ(xn) for any real numbers an , ..., a0 and any nonnegative number n .

Let f(x) and g(x) be functions from a set of real numbers to a set of real numbers.

Then

1. If $f(x)/g(x) = 0$, then f(x) is o(g(x)) . Note that if f(x) is o(g(x)), then f(x) is O(g(x)).

2. If $f(x)/g(x) = \infty$, then g(x) is o(f(x)) .

3. If $f(x)/g(x) < \infty$, then f(x) is θ(g(x)) .

4. If $f(x)/g(x) < \infty$, then f(x) is O(g(x)) .

For example,

(4x3 + 3x2 + 5)/(x4 – 3x3 – 5x -4)

= (4/x + 3/x2 + 5/x4)/(1 - 3/x - 5/x3 - 4/x4) = 0 .

Hence

(4x3 + 3x2 + 5) is o(x4 - 3x3 - 5x - 4),

or equivalently, (x4 - 3x3 - 5x - 4) is ω(4x3 + 3x2 + 5) .

Let us see why these rules hold. Here we give a proof for 4. Others can be proven similarly.

Proof: Suppose $f(x)/g(x) = C_1 < \infty$.

By the definition of limit this means that

∀ε> 0, ∃n0 such that |f(x)/g(x) – C1| < ε whenever x > n0

Hence –ε < f(x)/g(x) – C1 < ε

Hence –ε +C1 < f(x)/g(x) < ε +C1

In particular f(x)/g(x) < ε +C1

Hence f(x) < (ε +C1)g(x)

Let C = ε +C1 , then f(x) < Cg(x) whenever x > n0 .

Since we are interested in non-negative functions f and g, this means that |f(x) | ≤C | g(x) |

Hence f(x) = O(g(x)) .

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